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Particle detectors in Rindler and Schwarzchild space–times

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Abstract. It is shown that a particle detector coupled to the derivative of a scalar quantum field can be viewed as a bona fide 'particle detector'. The response of such a detector is compared with that of a linearly coupled monopole (DeWitt) detector when both are placed in each of three situations: an isotropic particle bath, in Rindler space, and in Schwarzschild space. From their responses it is shown that there is a fundamental difference in how these two detectors behave. As a result, the use of particle detectors to support the close links between quantum field theory in non-Minkowskian spaces and thermal physics is shown to be in need of deeper consideration as is the general role of particle detectors in this theory.

1. Introduction

Over recent years, the work of DeWitt (1979), Unruh (1976), Candelas (1980) and others has demonstrated that the response of certain particle detector models supports the apparently close links between quantum fields in non-Minkowskian spaces and thermal physics (see Sciama *et al* 1981).

In particular, Unruh demonstrated that a simple omni-directional detector when placed at a fixed distance outside a black hole would respond as though immersed in a bath of thermal radiation. Also, it was shown by Unruh and DeWitt that a similar detector undergoing uniform acceleration would likewise respond as though immersed in a bath of thermal radiation. (Recently it has been found that, in this latter case, the bath is not isotropic. See Hinton *et al* (1982).)

In all the cases cited above, the interaction Lagrangian for the particle detector was a simple linear monopole interaction

$$L_{\text{int}} = c' m(x)\phi[x] \tag{1}$$

where $m(x)$ is the monopole moment of the detector (which has some quantised internal degree of freedom allowing it to be excited out of a ground state), $\phi[x]$ is the scalar field and c' a small coupling constant.

This is not the only form of coupling, however, which can be used to construct a particle detector. We shall see below that we may also use

$$L'_{\text{int}} = c m^{\mu}(x)\partial_{\mu}\phi[x] \tag{2}$$

with $\partial_{\mu} \equiv \partial/\partial x^{\mu}$, which we shall call a derivatively coupled detector. In this case $m^{\mu}(x)$ is a dipole moment which can be oriented spatially in a variety of ways by a co-moving observer with the detector.

Using a criterion introduced below, we shall see that not only is the derivatively coupled detector a bona fide particle detector, but further, with such a detector, nexus demonstrated by the works cited above is apparently broken.

2. Definition of a particle detector

Following the lead of Unruh and DeWitt, we shall define a particle detector to be a mathematical construct which registers the occupation number of any given mode. The detector described by (1) (which we will call a 'DeWitt detector') satisfies this definition since immersion of such a detector in an isotropic particle bath characterised by occupation number n_k gives a transition probability per unit detector time, $\mathscr{W}$, of (Birrell and Davies 1982)

$$\mathscr{W} = c'^2|\langle E|m(0)|E_0\rangle|^2 2^{2-n}\pi^{(3-n)/2}(E^2-m^2)^{(n-3)/2} n_{(E^2-m^2)^{1/2}}\theta(E-m)/\Gamma[(n-1)/2)] \tag{3}$$

where n is the dimension of the (Minkowski) space–time, m is the mass of the field $\phi[x]$, and E labels the energy levels of the excited states of the detector. So, for a given energy level, the transition rate is proportional to the number of quanta in the mode of interest.

We now repeat this calculation, with some detail, for the derivatively coupled detector. We shall follow the approach utilised by Birrell and Davies (1982). To first-order perturbation theory, the transition amplitude for a transition of the $\phi[x]$ field from (vacuum) state $|\psi_0\rangle$ to excited state $|\psi\rangle$ and similarly for the detector $m^\mu(\tau)$ from (ground) state $|E_0\rangle$ to excited state $|E\rangle$ for a given trajectory $x(\tau)$ of the detector (τ being the detector's proper time) is

$$\mathrm{i}c\langle E, \psi| \int_{-\infty}^{\infty} \mathrm{d}\tau\, m^\mu(\tau)\, \partial_\mu\phi[x(\tau)]|\psi_0, E_0\rangle.$$

We assume that the time evolution equation of the detector, in its own rest frame, is $m^\mu(\tau) = \mathrm{e}^{\mathrm{i}H_0\tau} m^\mu(0)\, \mathrm{e}^{-\mathrm{i}H_0\tau}$ where $H_0|E\rangle = E|E\rangle$. So, we may write for the transition amplitude

$$\mathrm{i}c\langle E|m^\mu(0)|E_0\rangle \int_{-\infty}^{\infty} \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}(E-E_0)\tau}\, \partial_\mu\langle\psi|\phi[x(\tau)]|\psi_0\rangle. \tag{4}$$

From this we can evaluate the transition probability, P, of the detector to an excited state $|E\rangle$.

$$P = c^2\langle E|m^\mu(0)|E_0\rangle^*\langle E|m^\nu(0)|E_0\rangle\mathscr{F}(E-E_0)_{\mu\nu}$$

where

$$\mathscr{F}(E)_{\mu\nu} = \int_{-\infty}^{\infty}\mathrm{d}\tau\int_{-\infty}^{\infty}\mathrm{d}\tau'\, \mathrm{e}^{-\mathrm{i}E(\tau-\tau')}\, \partial_\mu\, \partial'_\nu\langle\psi_0|\phi[x(\tau)]\phi[x(\tau')]|\psi_0\rangle \tag{5}$$

where $\partial'_\nu \equiv \partial/\partial x'^\nu$ and $F_{\mu\nu}$ is called the response function. Note that $F_{\mu\nu}$ is not a tensor; the μ, ν refer to directions in the detector's rest frame. If $|\psi_0\rangle$ is taken to be a vacuum state, such as the Minkowski vacuum $|0_M\rangle$, then $\langle\psi_0|\phi[x(\tau)]\phi[x(\tau)]|\psi_0\rangle$ is replaced by the (appropriate) positive frequency Wightman Green function $G^+(x, x')$. Using (4) or (5) to evaluate the response of this detector to an isotropic particle bath characterised by n_k as above, we find for the response functions per unit time

$$\mathscr{F}(E)_{00}/T = 2^{2-n}\pi^{(3-n)/2}(E^2-m^2)^{(n-1)/2}n_{(E^2-m^2)^{1/2}}\theta(E-m)/\Gamma[(n-1)/2],$$

$$\mathscr{F}(E)_{ij}/T = \tfrac{1}{2}(2\pi)^{2-n}(E^2-m^2)^{(n-1)/2}n_{(E^2-m^2)^{1/2}}\theta(E-m)\int \mathrm{d}\Omega \cos\theta_i \cos\theta_j, \tag{6}$$

$$\mathscr{F}(E)_{0i}/T = \tfrac{1}{2}(2\pi)^{2-n}(E^2-m^2)^{(n-1)/2}n_{(E^2-m^2)^{1/2}}\theta(E-m)\int \mathrm{d}\Omega \cos\theta_i,$$

where $\cos\theta_i$ is the angle between the space-like direction defined by $m^i(0)$ and the space-like component of the n-dimensional momentum vector, and the integral is over the $(n-2)$-sphere of directions in $(n-1)$-dimensional space. (For $n=2$ the angular integral is set to 2.)

From (6) we can see that as with the DeWitt detector, this detector's transition rate will be proportional to the occupation number for a given energy level. So, the derivatively coupled detector can be justifiably considered a 'particle detector'.

3. Detectors in Rindler space

We now can compare the responses of the two detectors in Rindler space (i.e. undergoing uniform acceleration).

Firstly, we consider two-dimensional Rindler space, defined by Pfautsch (1981)

$$t = \xi \cosh\tilde{\tau}, \qquad z = \xi \sinh\tilde{\tau}, \tag{7}$$

and the proper time is $\tau = \xi\tilde{\tau}$. The appropriate Green function is (Birrell and Davies 1982)

$$G^+(x, x') = -(1/4\pi^2)\ln\{4\xi^2\sinh^2[(\Delta\tilde{\tau}/2)-(\mathrm{i}\varepsilon/2\xi)]\} \tag{8}$$

where $\Delta\tilde{\tau} = \tilde{\tau} - \tilde{\tau}'$. We find for the DeWitt detector

$$\mathscr{W} = c'^2|\langle E|m(0)|E_0\rangle|^2/(E-E_0)(\mathrm{e}^{(E-E_0)/kT}-1) \tag{9}$$

where $kT = 1/2\pi\xi$, which is identical to the response of such a detector in a thermal bath of radiation at the appropriate temperature. In all cases the transition rate is per unit detector time. Repeating this calculation for the derivatively coupled detector by using (5), we find

$$\mathscr{W}_{00} = c^2|\langle E|m^0(0)|E_0\rangle|^2(E-E_0)/(\mathrm{e}^{(E-E_0)/kT}-1), \tag{10}$$

$$\mathscr{W}_{11} = c^2|\langle E|m^1(0)|E_0\rangle|^2(E-E_0)/(\mathrm{e}^{(E-E_0)/kT}-1), \tag{11}$$

$$\mathscr{W}_{01} = \mathscr{W}_{10} = 0,$$

where in (10) we have coupled the detector to the proper time derivative of the field and in (11) to the local space derivative of the field (i.e. ∂_ξ).

From (6) we see that in the two-dimensional case, a derivative detector in Rindler space responds as though it were in a thermal bath of radiation with $T = 1/2\pi\xi k$. So, the DeWitt and derivatively coupled detectors agree in this situation. However, if we now consider the four-dimensional Rindler space we find that this concurrence does not carry over.

To consider the four-dimensional case, it is more convenient to use (4) rather than the Green function approach of (5). The scalar field equation in Rindler coordinates can be written as (Pfautsch 1981)

$$\left[\frac{1}{\xi^2}\frac{\partial^2}{\partial\tilde{\tau}^2}-\frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi}-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right]\phi=0$$

with mode functions

$$\phi_{\mathrm{R}}(x,\tilde{\omega},k_1,k_2)=(\sinh\pi\tilde{\omega})^{1/2}\,\mathrm{e}^{-\mathrm{i}\tilde{\omega}\tilde{\tau}}\,\mathrm{e}^{\mathrm{i}(k_1x+k_2y)}K_{\mathrm{i}\tilde{\omega}}(Q\xi)/2\pi^2 \tag{12}$$

where $K_{\mathrm{i}\tilde{\omega}}(Q\xi)$ is a Macdonald function, $Q^2=k_1^2+k_2^2$ and $\tilde{\omega}>0$. Expanding ϕ in terms of these Rindler modes and Rindler creation and annihilation operators, a_{R}^+, a_{R} and using the Bogoliubov transformation to Minkowski operators a_{M}^+, a_{M}

$$a_{\mathrm{R}}(\tilde{\omega},k_1,k_2)=\int \mathrm{d}^3k'\,[2\pi\omega'(1-\mathrm{e}^{-2\pi\tilde{\omega}})]^{-1/2}[(\omega'+k_3')/Q]^{\mathrm{i}\tilde{\omega}}\delta(k_1-k_1')\delta(k_2-k_2')$$
$$\times[a_{\mathrm{M}}(k_1',k_2',k_3')+\mathrm{e}^{-\pi\tilde{\omega}}a_{\mathrm{M}}^+(k_1',k_2',k_3')]$$

where $\omega'^2=k_1'^2+k_2'^2+k_3'^2$, we obtain for the DeWitt detector a transition rate per unit detector time

$$\mathscr{W}=c'^2|\langle E|m(0)|E_0\rangle|^2(E-E_0)/2\pi(\mathrm{e}^{(E-E_0)/kT}-1) \tag{13}$$

with $kT=1/2\pi\xi$ as usual. In evaluating (13) we have discarded a logarithmic divergence characteristic to these calculations since the detector is perceiving a constant flux of radiation over all proper time τ.

Repeating these calculations using (4) we find, after discarding an identical logarithmic divergence, that a detector coupled to the (proper) time derivative of the field has a transition rate of

$$\mathscr{W}_{00}=c^2|\langle E|m^0(0)|E_0\rangle|^2(E-E_0)^2/2\pi(\mathrm{e}^{(E-E_0)/kT}-1). \tag{14}$$

For a detector coupled to a space-like direction perpendicular to the direction of acceleration (i.e. the x or y directions), the transition rate is

$$\mathscr{W}_{ii}=c^2|\langle E|m^i(0)|E_0\rangle|^2(E-E_0)[1+(E-E_0)^2\xi^2]/6\pi\xi^2(\mathrm{e}^{(E-E_0)/kT}-1),\qquad i=1,2. \tag{15}$$

For a detector coupled to the space-like direction parallel to the direction of acceleration (i.e. locally the ξ direction)

$$\mathscr{W}_{33}=c^2|\langle E|m^3(0)|E_0\rangle|^2(E-E_0)(4+(E-E_0)^2\xi^2)/3\pi\xi^2(\mathrm{e}^{(E-E_0)/kT}-1). \tag{16}$$

Finally, there is now a cross term which appears if the detector is coupled to both the τ and ξ derivatives (i.e. local time and direction of acceleration derivatives). This term is

$$\mathscr{W}_{03}+\mathscr{W}_{30}=c^2\,\mathrm{Im}(\langle E|m^3(0)|E_0\rangle^*\langle E|m^0(0)|E_0\rangle)(E-E_0)^2/2\pi\xi(\mathrm{e}^{(E-E_0)/kT}-1). \tag{17}$$

In (14) and (17) we have $kT=1/2\pi\xi$. Referring to (6) we can see that only for coupling to the τ derivative does this detector give a thermal response. Furthermore, for such a detector which couples to space-like derivatives, different orientations of the detector will give different (non-Planckian) responses.

4. Detectors in Schwarzschild space

We find a similar situation to the above occurring for these two detectors placed in Schwarzschild space-times. We use the Hartle–Hawking Wightman Green function, which for a two-dimensional black hole is (Birrell and Davies 1982) $G_{\rm H}^{+} = -(1/4\pi^2)\ln[(\Delta\bar{u} - i\varepsilon)(\Delta\bar{v} - i\varepsilon)]$, where

$$\bar{u} = -e^{-Ku}/K, \qquad \bar{v} = e^{Kv}/K$$

with $K = 1/4M$, the surface gravity of the black hole, and

$$u = t - r^*, \qquad v = t + r^*, \qquad r^* = r + 2M\ln[(r/2M) - 1].$$

The Hartle–Hawking vacuum is defined with respect to the u, v coordinates, hence we find $G_{\rm H}^{+}(x, x') = -(1/4\pi^2)\ln\{4e^{2r^*/K}\sinh^2[(\Delta t/2K) - i\varepsilon]/K^2\}$ which in turn gives for the DeWitt detector a transition rate per unit proper time of (Birrell and Davies 1982)

$$\mathcal{W} = c'^2|\langle E|m(0)|E_0\rangle|^2/(E - E_0)(e^{(E-E_0)/kT} - 1) \tag{18}$$

where

$$kT = [64\pi^2 M^2(1 - 2M/r)]^{-1/2}. \tag{19}$$

Using (5) we find for the derivative detector

$$\mathcal{W}_{00} = c^2|\langle E|m^0(0)|E_0\rangle|^2(E - E_0)/e^{(E-E_0)/kT} - 1), \tag{20}$$

$$\mathcal{W}_{11} = c^2|\langle E|m^1(0)|E_0\rangle|^2(E - E_0)/(e^{(E-E_0)/kT} - 1). \tag{21}$$

$$\mathcal{W}_{10} = \mathcal{W}_{01} = 0,$$

with kT given by (19).

As with the two-dimensional Rindler space, both detectors give a thermal response. We now consider the four-dimensional Schwarzschild space. In this case the appropriate Wightman Green function is (Candelas 1980)

$$G_{\rm H}^{+}(x, x') = \sum_{l,m}\int_{-\infty}^{\infty} d\omega[e^{-i\omega(t-t')}Y_{lm}(\theta, \varphi)Y_{lm}^{*}(\theta', \varphi')\vec{R}_l(\omega|r)/\vec{R}_l^{*}(\omega|r')/(1 - e^{-2\pi\omega/K})$$
$$+ e^{i\omega(t-t')}Y_{lm}^{*}(\theta, \varphi)Y_{lm}(\theta', \varphi')\tilde{R}_l^{*}(\omega|r)\tilde{R}_l(\omega|r')/(e^{2\pi\omega/K} - 1)]/4\pi\omega \tag{22}$$

which for the DeWitt detector gives for the transition rates per unit proper time in the asymptotic regions

$$\mathcal{W} = c'^2|\langle E|m(0)|E_0\rangle|^2(E - E_0)/2\pi(e^{(E-E_0)/kT} - 1), \qquad r \to 2M, \tag{23}$$

$$\mathcal{W} = c'^2|\langle E|m(0)|E_0\rangle|^2(E - E_0)/2\pi(e^{(E-E_0)/kT} - 1), \qquad r \to \infty, \tag{24}$$

with kT given in (19).

Repeating for the derivatively coupled detector we find

$$\mathcal{W}_{00} = c^2|\langle E|m^0(0)|E_0\rangle|^2(E - E_0)^3/2\pi(e^{(E-E_0)/kT} - 1), \qquad r \to 2M, \tag{25}$$

$$\mathcal{W}_{00} = c^2|\langle E|m^0(0)|E_0\rangle|^2(E - E_0)^3/2\pi(e^{(E-E_0)/kT} - 1), \qquad r \to \infty. \tag{26}$$

To evaluate the radial-derivative component of the responses, we adopt the procedure used by Candelas (1980), and find (see the appendix)

$$\frac{\partial^2}{\partial r^*\partial r^{*\prime}}\sum_{l=0}^{\infty}(2l+1)\vec{R}_l(\omega|r)\vec{R}_l^{*}(\omega|r')|_{r=r'} \underset{r\to 2M}{\sim} (1 + 16M^2\omega^2)\omega^2/3M^2(1 - 2M/r)$$

and

$$\frac{\partial^2}{\partial r^* \partial r^{*\prime}} \sum_{l=0}^{\infty} (2l+1)\tilde{R}_l^*(\omega|r)\tilde{R}_l(\omega|r')|_{r=r'} \underset{r\to\infty}{\sim} \tfrac{4}{3}\omega^4$$

giving

$$\mathcal{W}_{11} = c^2|\langle E|m^1(0)|E_0\rangle|^2(E-E_0)/24\pi M^2(e^{(E-E_0)/kT}-1), \qquad r\to 2M, \tag{27}$$

$$\mathcal{W}_{11} = c^2|\langle E|m^1(0)|E_0\rangle|^2(E-E_0)^3/6\pi(e^{(E-E_0)/kT}-1), \qquad r\to\infty. \tag{28}$$

We now consider the angular-derivative transition rates. From (22) the quantities we are interested in are

$$\frac{\partial^2}{\partial\theta\,\partial\theta'} \sum_{m=-l}^{l} Y_{ml}^*(\theta,\varphi)Y_{ml}(\theta',\varphi)|_{\theta=\theta'} = \frac{2l+1}{4\pi}\frac{\partial P_l(x)}{\partial x}\bigg|_{x=1} = l(l+1)(2l+1)/8\pi \tag{29}$$

$$\sin^{-2}\theta \frac{\partial^2}{\partial\varphi\,\partial\varphi'} \sum_{m=-l}^{l} Y_{ml}^*(\theta,\varphi)Y_{ml}(\theta,\varphi')|_{\varphi=\varphi'} = l(l+1)(2l+1)/8\pi. \tag{30}$$

We find (see the appendix)

$$r^{-2}\sum_{l=0}^{\infty} l(l+1)(2l+1)|\vec{R}_l(\omega|r)|^2 \underset{r\to 2M}{\sim} 6(1+16M^2\omega^2)\omega^2/M^2(1-2M/r)^2, \tag{31}$$

$$r^{-2}\sum_{l=0}^{\infty} l(l+1)(2l+1)|\vec{R}_l(\omega|r)|^2 \underset{r\to\infty}{\sim} \tfrac{8}{3}\omega^4. \tag{32}$$

The factor of $\sin^{-2}\theta$ in (30) occurs due to the coordinate singularities which occur at the poles of the spherical coordinate system. By spherical symmetry, we can see that (29) and (30) are, in fact, making the same statement, hence we only need consider the θ-derivative component of the response. From (31) and (32) we have

$$r^{-2}\mathcal{W}_{22} = c^2|\langle E|m^2(0)|E_0\rangle|^2(E-E_0)/96\pi M^2(1-2M/r)(e^{(E-E_0)/kT}-1), \qquad r\to 2M, \tag{33}$$

$$r^{-2}\mathcal{W}_{22} = c^2|\langle E|m^2(0)|E_0\rangle|^2(E-E_0)^3/6\pi(e^{(E-E_0)/kT}-1), \qquad r\to\infty, \tag{34}$$

and we also have

$$(r\sin\theta)^{-2}\mathcal{W}_{33} = r^{-2}\mathcal{W}_{22}. \tag{35}$$

Finally, we evaluate the cross-terms. Since

$$\frac{\partial}{\partial\theta}\sum_{m=-l}^{l} Y_{ml}^*(\theta,\varphi)Y_{ml}(\theta',\varphi)|_{\theta=\theta'} = \frac{\partial}{\partial\varphi}\sum_{m=-l}^{l} Y_{ml}^*(\theta,\varphi)Y_{ml}(\theta,\varphi')|_{\varphi=\varphi'} = 0$$

we have no cross-terms involving θ or φ derivatives. However, we find we do have a cross-term involving the t and r^* derivatives. Again, following the same approach as before, we find for the total cross-term,

$$\mathcal{W}_{01} = \mathcal{W}_{10} = 0, \qquad r\to\infty, \tag{36}$$

$$\mathcal{W}_{01} + \mathcal{W}_{10} = c^2(E-E_0)^2\,\mathrm{Im}(\langle E|m^1(0)|E_0\rangle^* \times \langle E|m^0(0)|E_0\rangle)/4\pi M(e^{(E-E_0)/kT}-1), \qquad r\to 2M. \tag{37}$$

5. Conclusions

Before proceeding with any conclusions, it should be noted that the derivatively coupled detector is not a directional detector in the sense of the 'bi-cone' detector introduced in Hinton *et al* (1982). Although the derivatively coupled detector has a directionality in its coupling to the field, it is still receptive to the full two-sphere of directions of modes in momentum space. By contrast, the bi-cone is shielded from some of these mode directions. Hence, just as the DeWitt detector effectively responds to the average of the modes over the two-sphere of directions, so does this derivative detector similarly take such an average. (If we did restrict the directions in momentum space accessible to the derivatively coupled detector—i.e. a derivatively coupled bi-cone detector—we would find, in Rindler space, a similar directional dependence as for the bi-cone.)

With this in mind, we can now see that just as the DeWitt detector is omni-directional, so is the derivatively coupled detector, once we are given the component of the derivative of the field to which it is coupled. So, just as the linearly coupled detector responds as though it were in an isotropic bath of radiation of a given spectrum n_k, we can make a similar statement about the derivatively coupled detector. Furthermore, using (5) and (6), we can compare these perceived baths of radiation.

Using these two equations, we find the following:

(i) in two-dimensional space–times, both detectors agree as to what they observe in all three situations (i.e. isothermal bath, Rindler space and Schwarzschild space). However, in four dimensions,

(ii) the DeWitt detector and time-like-derivatively coupled detector agree in all three situations;

(iii) the DeWitt detector and space-like-derivatively coupled detector do not agree in all three situations;

(iv) in the Schwarzschild space–time, in the region $r \to \infty$, both detectors respond to the Hartle–Hawking vacuum as though immersed in an isotropic bath of Planckian radiation. Moving toward the event horizon ($r = 2M$), the DeWitt detector responds as though in a bath of Planckian radiation of ever increasing temperature. On the other hand, the derivatively coupled detector perceives non-Planckian radiation.

(v) Comparing the time-like and radial components ($\mathscr{W}_{00}$, $\mathscr{W}_{11}$ and $\mathscr{W}_{10}$) with the angular components ($\mathscr{W}_{22}$ and $\mathscr{W}_{33}$) of the derivatively coupled detector's response, we note that although the former diverge as $r \to 2M$ due to the Planck factor, the latter diverge more vigorously due to the presence of the extra factor, $(1-2M/r)^{-1}$. In addition, from (6) we can also see that

(vi) given the different components of the derivative of the field to which the derivatively coupled detector can be coupled, the response of this detector is orientation dependent.

Accepting that both detectors are bona fide 'particle detectors', we are now faced with the dilemma of two detectors giving different information when placed in identical situations. To reconcile this, we must introduce the concept of 'particle detector equivalence' which will be the topic of a forthcoming paper. Also, we must look in greater detail at how these detectors work. For example, it is known from the Bogoliubov transformations that the Minkowski vacuum is a thermal state of finite temperature with respect to the Fulling (i.e. Rindler space) vacuum (Pfautsch 1981). Although the Bogoliubov coefficients are directionally dependent, the linearly coupled detector responds as though it is an isotropic thermal state and the derivatively coupled detector need not even perceive it as Planckian. To appreciate the reasons for this difference we must look at how these detectors average over Bogoliubov transformations.

Obviously there are not only other ways of coupling to the scalar field (e.g. $m(x)\phi(x)^2$) but also a variety of interaction Lagrangians available to us when detecting higher spin fields. Further, we could relax our definition of a particle detector by only requiring that its response uniquely determine the particle spectrum n_k. Thus we see that the question of particle detector 'equivalence' spans not only the various detectors of a given quantum field, but also those of other fields.

Finally, there is also the question of extended detectors. In the above we only considered infinitesimal point detectors, and the extension of these results to extended detectors may be far from trivial (Grove and Ottewill 1981).

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Appendix

We shall calculate the quantities

$$\frac{\partial^2}{\partial r^* \, \partial r^{*\prime}} \sum_{l=0}^{\infty} (2l+1) \overleftarrow{R}_l^*(\omega|r) \overleftarrow{R}_l(\omega|r')|_{r=r'} \qquad \text{for } r \to \infty, \tag{A1}$$

$$\frac{\partial^2}{\partial r^* \partial r^{*\prime}} \sum_{l=0}^{\infty} (2l+1) \vec{R}_l(\omega|r) \vec{R}_l^*(\omega|r')|_{r=r'} \qquad \text{for } r \to 2M. \tag{A2}$$

Following the approach of Candelas (1980), we note that we can write

$$\frac{\partial^2}{\partial r \, \partial r'} G_{\mathrm{B}}^{+}(t, r, \theta, \varphi; 0, r'\theta, \varphi)|_{r=r'}$$

$$= \int_0^{\infty} \mathrm{d}\omega \, \mathrm{e}^{-\mathrm{i}\omega t} \frac{\partial^2}{\partial r \, \partial r'} \sum_{l=0}^{\infty} (2l+1) [\vec{R}_l(\omega|r) \vec{R}_l^*(\omega|r')$$

$$+ \overleftarrow{R}_l^*(\omega|r) \overleftarrow{R}_l(\omega|r')]/16\pi^2\omega|_{r=r'} \underset{r\to\infty}{\sim} 1/2\pi^2 t^4.$$

Using

$$\int_0^{\infty} \mathrm{d}\omega \, \omega^3 \, \mathrm{e}^{-\mathrm{i}\omega t} = 6/t^4$$

and

$$(\mathrm{d}r/\mathrm{d}r^*) = (\mathrm{d}r^*/\mathrm{d}r)^{-1} = (1 - 2M/r)$$

we find

$$(\partial^2/\partial r^* \, \partial r^{*\prime}) \sum_{l=0}^{\infty} (2l+1) \overleftarrow{R}_l^*(\omega|r) \overleftarrow{R}_l(\omega|r')|_{r=r'} \underset{r\to\infty}{\sim} \tfrac{4}{3}\omega^4. \tag{A3}$$

For (A2) we use the result that to leading order we have

$$\sum_{l=0}^{\infty} (2l+1)\vec{R}_l(\omega|r)\vec{R}_l^*(\omega|r') \underset{r\to 2M}{\sim} (2/M^2\Gamma(\mathrm{i}q)\Gamma(-\mathrm{i}q)) \int_0^\infty \mathrm{d}l\, l K_{\mathrm{i}q}(2l\alpha^{1/2})K_{\mathrm{i}q}(2l\alpha'^{1/2})$$

where $q=4M\omega$ and $\alpha=(r/2M)-1$. By use of the chain rule, the quantity we desire is

$$\int_0^\infty \mathrm{d}l\, l \frac{\partial}{\partial\alpha} K_{\mathrm{i}q}(2l\alpha^{1/2}) \frac{\partial}{\partial\alpha'} K_{\mathrm{i}q}(2l\alpha'^{1/2})|_{\alpha=\alpha'}$$

$$= (1/4\alpha)\int_0^\infty \mathrm{d}l\, l^3[K_{\mathrm{i}q+1}(2l\alpha^{1/2})+K_{\mathrm{i}q-1}(2l\alpha^{1/2})]^2$$

$$= (4+q^2)\Gamma(1+\mathrm{i}q)\Gamma(1-\mathrm{i}q)/96\alpha^3$$

which gives

$$\frac{\partial^2}{\partial r^*\,\partial r^{*\prime}} \sum_{l=0}^{\infty} (2l+1)\vec{R}_l(\omega|r)\vec{R}_l^*(\omega|r')|_{r=r'} \underset{r\to 2M}{\sim} (1+16M^2\omega^2)\omega^2/3M^2(1-2M/r). \qquad \text{(A4)}$$

Next, we evaluate the angular derivatives of the Hartle–Hawking Green function. The quantities we seek are

$$(\partial^2/r^2\partial\theta\,\partial\theta')G_{\mathrm{H}}^{+}(t,r,\theta,\varphi;t',r,\theta',\varphi)|_{\theta=\theta'}$$

and

$$(\partial^2/r^2\sin^2\theta\,\partial\varphi\,\partial\varphi')G_{\mathrm{H}}^{+}(t,r,\theta,\varphi;t',r,\theta,\varphi')|_{\varphi=\varphi'}.$$

Using (22) and

$$(\partial^2/\partial\theta\,\partial\theta')\sum_{m=-l}^{l} Y_{ml}^*(\theta,\varphi)Y_{ml}(\theta',\varphi)|_{\theta=\theta'} = l(l+1)(2l+1)/8\pi$$

we see we must evaluate

$$\sum_{l=0}^{\infty} l(l+1)(2l+1)|\bar{R}_l(\omega|r)|^2 \qquad \text{as } r\to\infty,$$

$$\sum_{l=0}^{\infty} l(l+1)(2l+1)|\vec{R}_l(\omega|r)|^2 \qquad \text{as } r\to 2M.$$

Using spherical symmetry, we can write

$$(\partial^2/r^2\partial\theta\partial\theta')G_{\mathrm{B}}^{+}(t,r,\theta,\varphi;0,r,\theta',\varphi)|_{\theta=\theta'} \underset{r\to\infty}{\sim} 1/2\pi^2t^4;$$

hence, following exactly the same approach as above, we get

$$r^{-2}\sum_{l=0}^{\infty} l(l+1)(2l+1)|\bar{R}_l(\omega|r)|^2 \underset{r\to\infty}{\sim} \tfrac{8}{3}\omega^4. \qquad \text{(A5)}$$

For the other limit, we have, to leading order

$$r^{-2}\sum_{l=0}^{\infty} l(l+1)(2l+1)|\vec{R}_l(\omega|r)|^2 \underset{r\to 2M}{\sim} (2M^4\Gamma(\mathrm{i}q)\Gamma(-\mathrm{i}q))^{-1}\int_0^\infty \mathrm{d}l\, l^3K_{\mathrm{i}q}^2(2l\alpha^{1/2})$$

$$= 2(1+16M^2\omega^2)\omega^2/3M^2(1-2M/r)^2. \qquad \text{(A6)}$$

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